A universal parameterization of finite-size effects for interfaces in two dimensions, conformal mappings and local scale invariance

1992 J. Phys. A: Math. Gen. 25 L685

(http://iopscience.iop.org/0305-4470/25/11/012)

J. Phys. A: Math. Gen. 25 (1992) L685–L691. Printed in the UK

LETTER TO THE EDITOR

A universal parametrization of finite-size effects for interfaces in two dimensions, conformal mappings and local scale invariance

A O Parry†

H H Wills Physics Laboratory, University of Bristol, Bristol BS8 1TL, UK

Received 17 January 1992, in final form 27 March 1992

Abstract. We show that all equilibrium statistical properties of an interface confined in a strip geometry, with arbitrary aspect ratio, exactly at a second-order, fluctuation dominated, interfacial unbinding (wetting) transition are determined by a single number q for interfacial binding potentials that are conformally mapped from the semi-infinite plane. The parameter q distinguishes the fluctuation regimes describing the wetting transition and can be directly related to wetting critical exponents. In the strong-fluctuation regime we find $q=0$ whilst in the weak-fluctuation regime $q=1$. The values of q at all intermediate-fluctuation scaling regimes are also determined. We show that the eigenstates of the transfer differential operator for conformally mapped marginal long-ranged potentials are the simplest possible generalization of the eigenstates corresponding to systems with short-ranged forces. We speculate that the universal parametrization of the finite-size effects at fluctuation dominated wetting transitions is a consequence of local scale invariance.

In this letter we report some striking finite-size (FS) effects associated with the confinement of an interface separating two 'bulk' fluid (or Ising spin) phases in a two-dimensional strip of length M $(0\leqslant x\leqslant M)$ and width L $(0\leqslant y\leqslant L)$. The fluctuations of the interface are described by the continuum effective interfacial Hamiltonian [1, 2]

$$H[y(x)]=\int_0^M \mathrm{d}x\left\{\frac{\Sigma}{2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+V(y(x))\right\} \tag{1}$$

where Σ is the surface stiffness coefficient, $V(y)$ is the interfacial binding potential and the (single-valued) function $y(x)$ describes the distance of the interface from the wall at $y=0$. The boundary conditions $y(0)$, $y(M)$ may be regarded as fixed, free or periodic. More specifically we shall consider the FS effects exactly at the critical wetting temperature of each surface [1–3]. The binding potential $V(y)$ is chosen to be symmetric about $L/2$ so that each wall is wet by a different bulk phase. FS effects in such geometries particularly for systems with pure short-ranged forces (i.e. Ising-like spin systems) and $M=\infty$ have received some considerable attention in the recent literature [4]. The behaviour of the fluid phase in such geometries exhibits interesting fluctuation induced effects due to the thermal wandering of the intrinsic interface. In the present letter we

† Permanent address after 1 August 1992: Department of Mathematics, Imperial College, London SW7 2BZ, UK.

shall establish that all thermodynamic and statistical averages for the confined interface described by (1) are characterized by a single number (for fixed temperature T and Σ) q in all fluctuation-dominated regimes [2, 5] for $V(y)$ obtained by a conformal mapping of the potential $V_\infty(y)$ corresponding to the semi-infinite plane at the appropriate wetting transition. Confinement in the strong-fluctuation (SFL) regime forms a universality class of FS effects described by $q=0$, whilst confinement in the weak-fluctuation (WFL) regime forms a universality class corresponding to $q=1$. In the intermediate-fluctuation (IFL) regimes we find $q=(1-\sqrt{1+8\tilde{w}\beta^2\Sigma})/2$ for the SFL/WFL borderline whilst $q=(1+\sqrt{1+8\tilde{w}\beta^2\Sigma})/2$ for the WFL/MF (mean-field) case. Here $\tilde{w}$ plays the role of a generalized Hamaker constant to be defined below and $\beta=1/k_BT$. We show that the eigenfunctions (and hence eigenvalues) of the transfer integral operator of (1), for a particular class of potentials (which correspond to different scaling regimes) are universal functions of elementary trigonometric functions characterized by *the number* q *(the fluctuation designator) and is directly related to the* critical exponents describing the wetting transition in the semi-infinite geometry ($L=\infty$, $M=\infty$). We speculate that the striking simplicity of this universal parametrization is a consequence of local scale invariance [6] at fluctuation-dominated wetting transitions in $d=2$.

To begin recall that all thermodynamic averages (i.e. the free-energy, probability distributions, correlation functions) for the Hamiltonian (1) with arbitrary $V(y)$ and L and M can be written in terms of the eigenfunctions $\psi_n(y)$ and eigenvalues E_n of the Schrödinger operator [2]

$$\left(-\frac{1}{2\beta^2\Sigma}\frac{\mathrm{d}^2}{\mathrm{d}y^2}+V(y)-E_n\right)\psi_n(y)=0. \tag{2}$$

Hereafter we set $2\beta^2\Sigma\equiv 1$ without any loss of generality. Since we are interested in the universal scaling properties of the FS effects we shall simply ignore all irrelevant operators appearing in $V(y)$. The FS effects in the various scaling regimes are easily determined.

1. FS effects in the SFL regime

Consider the semi-infinite plane $y>0$, $M=\infty$. All (single wall) binding potentials $V_\infty(y)$ that decay as $V(y)\sim y^{-r}$ with $r>2$ exhibit universal critical wetting critical behaviour described by the SFL regime [7]. The mean interface displacement diverges as $\langle y\rangle\sim t^{-\beta_s}$ on approaching the wetting temperature T_w ($t\equiv(T_w-T)/T_w$) with $\beta_s=1$. Similarly the perpendicular correlation length $\xi_\perp\sim t^{-\nu_\perp}$ with $\nu_\perp=1$. The transverse correlation length diverges as $\xi_\parallel\sim t^{-\nu_\parallel}$ with $\nu_\parallel=2$, while the probability distribution $P(y)$, corresponding to the probability of finding the interface at height y, varies as $P(y)/P(1)\sim y^{(\theta-1)}$ with $\theta=1$ exactly at $t=0$ [8]. Here θ is the short-distance expansion (SDE) critical exponent for the order parameter profile $m(y)$ [8, 9]. In zero bulk field the order parameter profile exhibits the algebraic decay; $m(y)-m_0\approx(y/\xi_\perp)^\theta$ at short distances $0\ll y\ll\xi_\perp$. Here m_0 is the bulk order parameter of the phase adsorbed at the wall. The scaling properties of correlation functions in the SFL may be found by solving the Schrödinger equation (2) with $V_\infty(y)=0$ $\forall y>0$ subject to the Cauchy conditions [10] $\psi_n'(0)\propto -t\psi_n(0)$. For confinement exactly at the critical wetting temperature we choose $V(y)=0$ $\forall 0<y<L$ and impose the Neumann boundary conditions $\psi_n'(0)=\psi_n'(L)=0$. The eigenfunctions and eigenvalues then follow as (see also [4, third

reference])

$$\psi_n(y) \propto \cos\frac{n\pi y}{L} \qquad n=0,1,2,3,\ldots \qquad (\text{SFL})$$
$$E_n = \frac{n^2\pi^2}{L^2} \qquad (\text{SFL}) \tag{3}$$

where we have omitted the trivial normalization factor. These suffice to determine all the statistical properties of the interface confined to the finite length M and width L strip. Clearly the FS effects at the SFL regime described by (3) form a universality class provided any long-ranged forces appearing in $V(y)$ remain irrelevant.

II. FS *effects in the* WFL *regime*

When the asymptotic decay of $V_\infty(y)$ for the semi-infinite plane is $V_\infty(y) = -ty^{-r}(y\to\infty)$ with $r<2$ the critical wetting transition belongs to the WFL scaling regime [2, 5]. The operator y^{-r} is relevant in the renormalization group (RG) sense and hence its coefficient is a linear measure of the deviation from criticality. In the WFL scaling regime the critical exponents are not universal. They are given by [5] $\beta_s = \nu_\perp = \nu_\parallel/2 = 1/(\tau - r)$ with $\tau = 2(d-1)/(3-d) = 2$. The rescaled probability distribution exactly at criticality $P(y)/P(1) \propto y^{\theta-1}$ with $\theta = 3$ $\forall r<2$ [9]. To specify the criticality in the WFL regime we simply set $t=0$ and ignore irrelevant operators. For the strip geometry the FS effects at WFL regime critical wetting transitions therefore form a universality class $\forall r<2$. The critical scaling behaviour can be modelled by the (trivial) potential $V(y)=0$ $\forall 0<y<L$ with the Dirichlet boundary conditions $\psi_n(0) = \psi_n(L) = 0$. The eigenfunctions and eigenvalues are given by

$$\psi_n(y) \propto \sin\frac{(n+1)\pi y}{L} \qquad n=0,1,2,3 \qquad (\text{WFL})$$
$$E_n = \frac{(n+1)^2\pi^2}{L^2} \qquad (\text{WFL}) \tag{4}$$

corresponding to the standard quantum mechanical problem of a particle in an infinite square well [4].

III. FS *effects at the* WFL/MF *boundary*

In general, the single wall binding potential $V_\infty(y)$ describing the unbinding of the interface at a critical wetting transition has a decay law specified by two exponents when $2>r>0$,

$$V_\infty(y) = -ty^{-r} + \tilde{w}y^{-r-1} \qquad y>0,\ \tilde{w}>0. \tag{5}$$

If we allow for such potentials the WFL regime described above corresponds to $1<r<2$ [2, 9]. For $r<1$ the second repulsive term in (5) is relevant and the critical exponents are mean-field-like. Such transitions are not fluctuation-dominated since $l \gg \xi_\perp$. We shall not consider these potentials here. The WFL/MF borderline corresponds to the case $r=1$ so that the repulsive operator in (5) is marginal. The length scale exponents are now given by $\beta_s = \nu_\perp = \nu_\parallel/2 = 1$. In this case the SDE exponent θ is non-universal, however, $\theta = 2+\sqrt{1+8\tilde{w}\beta^2\Sigma}$ [9]. Exactly at the wetting transition $t=0$, $V_\infty(y) = \tilde{w}y^{-2}$

for $y>0$. Recall that in the absence of irrelevant operators the form of $V_\infty(y)$, exactly at the SFL and WFL critical wetting transitions, is $V_\infty(y)=0\ \forall y>0$. Consequently the potential in the strip geometry that models the FS effects at these transitions is unambiguously defined, and is simply $V(y)=0,\ \forall 0<y<L$. In the *present* case the presence of a marginal long-ranged operator means that the choice of $V(y)$ in the strip is not obvious. For instance, one may choose a potential $V_s(y)=\tilde{w}(y^{-2}+(L-y)^{-2})$ corresponding to a superposition of the two marginal semi-infinite potentials. An alternative choice is to set $V(y)=V_c(y)$ where

$$V_c(y)=\frac{\tilde{w}\pi^2}{L^2}\left(\sin\frac{\pi y}{L}\right)^{-2}\qquad 0<y<L\qquad (\text{WFL/MF}). \tag{6}$$

V_c is obtained [6] by conformally mapping the semi-infinite potential $V_\infty(y)=\tilde{w}y^{-2}$, using the standard logarithmic function $w(z)=(L/\pi)\ln z$ with $w=x+iy$ (strip) and $z=x+iy$ (semi-infinite plane). The reason for this choice of $V(y)$ will become clear. We solve for $\psi_n(y)$ and E_n using Dirichlet boundary conditions $\psi_n(0)=\psi_n(L)=0$. For $n=0$ and even eigenstates

$$\psi_n(y)\propto\left(\sin\frac{\pi y}{L}\right)^{\frac{1}{2}(1-\sqrt{1+4\tilde{w}})}F\left(-\frac{n}{2}+\frac{(2-\sqrt{1+4\tilde{w}})}{2},\frac{n}{2}-\frac{1}{2};\frac{1}{2};\cos^2\frac{\pi y}{L}\right) \tag{7a}$$

whilst for odd states

$$\psi_n(y)\propto\left(\sin\frac{\pi y}{L}\right)^{\frac{1}{2}(1-\sqrt{1+4\tilde{w}})}\cos\frac{\pi y}{L}\,F\left(-\frac{n}{2}+\frac{(3-\sqrt{1+4\tilde{w}})}{2},\frac{n}{2}-\frac{3}{2};\frac{1}{2};\cos^2\frac{\pi y}{L}\right). \tag{7b}$$

Here $F(a,b;c;z)$ is the usual hypergeometric function and recall we have set $2\beta^2\Sigma\equiv1$.

The eigenvalues have a somewhat simpler form,

$$E_n=\left(\frac{1+\sqrt{1+4\tilde{w}}}{2}+n\right)^2\frac{\pi^2}{L^2}\qquad n=0,1,2,3,\ldots\qquad (\text{WFL/MF}). \tag{8}$$

Finite-size effects at the WFL/MF borderline may be extended to the regime $-\frac{1}{4}<\tilde{w}<0$ by introducing the boundary condition (for $y=0$) $(\mathrm{d}\ln\psi_n(y)/\mathrm{d}\ln y)|_{y=0}=(1+\sqrt{1+4\tilde{w}})/2$. This mimics the effect of a short-ranged operator in the potential $V(y)$ at the WFL/MF boundary. Equations (7) and (8) remain valid for this case.

IV. FS effects at the SFL/WFL boundary

When the binding potential $V_\infty(y)$ describing the unbinding transition in the semi-infinite plane has a marginal long-ranged tail $V_\infty(y)=\tilde{w}y^{-2}$ for $y\to\infty$ and a short-ranged term the critical wetting transition may belong to the SFL/WFL borderline. To specify the critical properties of the transition we must, in general, carefully specify the short-ranged properties of $V_\infty(y)$. A worked model [11] has shown the existence of three subregimes (A, B and C) each of which has different critical behaviour. We wish to study the FS effects exactly at the wetting transition in the presence of such a marginal operator at the SFL/WFL borderline. The choice of $V(y)$ in the strip is again crucial. We invoke, once more, the logarithmic mapping to generate $V(y)$ from $V_\infty(y)$. First we restrict ourselves to subregime (A) of the model of Lipowsky and Nieuwenhuizen [11]. This corresponds to the case (see also [7]) where the transition is determined solely by the long-ranged tail in $V_\infty(y)$ with $\tilde{w}=-\frac{1}{4}$ at criticality (the short-ranged term can be set to zero except infinitesimally close to the wall). For this transition $\beta_s = \nu_\perp = \nu_\parallel/2 = \infty$ corresponding to an essential singularity. The SDE critical exponent $\theta = 2$. Thus FS effects should be accounted for by

$$V_c(y) = -\frac{1}{4}\frac{\pi^2}{L^2}\left(\sin\frac{\pi y}{L}\right)^{-2} \tag{9}$$

with Dirichlet boundary conditions $\psi_n(0) = \psi_n(L) = 0$. The solutions may be obtained as in III. We quote only the result for the eigenvalues

$$E_n = \left(\frac{1}{2} + n\right)^2 \frac{\pi^2}{L^2} \qquad n = 0, 1, 2, 3, \ldots \qquad (\text{SFL/WFL})\ \text{A}. \tag{10a}$$

Results similar to (7*a*) and (7*b*) pertain for the eigenfunctions. For subregimes (B) we use the potential (6) but solve for the boundary conditions (at $y = 0$) $(\mathrm{d}\ln\psi_n(y)/\mathrm{d}\ln y)|_{y=0} = \frac{1}{2}(1 - \sqrt{1 + 4\tilde{w}})$ with $-\frac{1}{4} < \tilde{w} < 3/4$. Again we only quote the results for the eigenvalues

$$E_n = \left(\frac{1 - \sqrt{1 + 4\tilde{w}}}{2} + n\right)^2 \frac{\pi^2}{L^2} \qquad n = 0, 1, 2, 3, \ldots \qquad (\text{SFL/WFL})\ \text{B}. \tag{10b}$$

FS effects in subregime (C) will not concern us here since the wetting transition is first-order for this case [12].

As stated earlier the eigenfunctions and eigenvalues determine the complete statistical behaviour of the FS system in the respective universality class/scaling regimes. The analytic expressions we have derived for the eigenfunctions at criticality in the strip exhibit a remarkable 'universality' which is only manifest when the results are expanded in elementary trigonometric form. Namely, all the eigenfunctions and eigenvalues (for fixed T, Σ) for the *different* fluctuation scaling regimes have the same simple universal form determined by a single parameter q which distinguishes the universality classes and scaling regimes. For this reason we denote q the fluctuation designator. For the eigenvalues the general form is simple: $E_n = \varepsilon_n(q)$ where

$$\varepsilon_n(q) = (q + n)^2 \frac{\pi^2}{L^2} \qquad n = 0, 1, 2, 3, \ldots \qquad \text{all regimes} \tag{11}$$

which identifies $q = 0, 1, \frac{1}{2}(1 + \sqrt{1 + 4\tilde{w}})$ and $\frac{1}{2}(1 - \sqrt{1 + 4\tilde{w}})$ for the respective regime I–IV. The orthonormal eigenfunctions may be written

$$\psi_n(y) = S_n(y; q) \qquad \text{all regimes.} \tag{12}$$

The first five eigenfunctions $S_n(y; q)$ are given in table 1. The higher eigenfunctions are easily generated. For even n the generic form is $(\sin \pi y/L)^q \times$ (a polynomial of degree $n/2$ in $\sin^2 \pi y/L$) with an obvious generalization for n odd [13]. It follows that all the critical properties of the interface in the strip with arbitrary dimensions L and M are characterized by the single number q. All values of q lie on the parabola $\tilde{w}(q) = q^2 - q$ with the restriction $\tilde{w} < 3/4$ on the lower branch. Further it is clear from (11) and (12) that the structure of the eigenstates in the IFL for conformally mapped potentials is the simplest possible generalization of the eigenstates describing FS effects in the SFL and WFL regimes. Whilst the mathematical analysis used to establish this fact is elementary it should be emphasized that the existence of a simple universal parametrization of the eigenstates encompassing different universality classes and non-universal regimes is non-trivial. Recall that the incorporation of the WFL/MF and

Table 1. This table shows the first five universal eigenfunctions and eigenvalues which describe the FS effects in the SFL, WFL, WFL/MF and SFL/WFL regimes. These correspond to $q=0, 1$, and $(1\pm\sqrt{1+4\tilde{w}})/2$ respectively. The coefficients $c_n(q)$ are normalization factors.

n	$S_n(y;q)$	$\varepsilon_n(q)\dfrac{L^2}{\pi^2}$
0	$c_0(q)\left(\sin\dfrac{\pi y}{L}\right)^q$	q^2
1	$c_1(q)\left(\sin\dfrac{\pi y}{L}\right)^q\cos\dfrac{\pi y}{L}$	$(q+1)^2$
2	$c_2(q)\left(\sin\dfrac{\pi y}{L}\right)^q\left(1-\dfrac{2(q+1)}{2q+1}\sin^2\dfrac{\pi y}{L}\right)$	$(q+2)^2$
3	$c_3(q)\left(\sin\dfrac{\pi y}{L}\right)^q\left(1-\dfrac{2(q+2)}{2q+1}\sin^2\dfrac{\pi y}{L}\right)\cos\dfrac{\pi y}{L}$	$(q+3)^2$
4	$c_4(q)\left(\sin\dfrac{\pi y}{L}\right)^q\left(1-\dfrac{4(q+2)}{1+2q}\sin^2\dfrac{\pi y}{L}+\dfrac{4(q+2)}{1+2q}\dfrac{(q+3)}{(3+2q)}\sin^4\dfrac{\pi y}{L}\right)$	$(q+4)^2$

SFL/WFL borderlines into the superuniversal parametrizations (11) and (12) was achieved by means of a conformal mapping that constructed $V(y)$ from $V_\infty(y)$. The symmetric potential $V_s(y)$ introduced in III also describes an IFL regime but is *not* characterized by the universal eigenvalue and eigenfunction parametrization. Whilst the partition function for this potential is still characterized by q [13] the form of the eigenfunctions when $\tilde{w}\neq 0$ bares no similarity to case $\tilde{w}=0$. Consequently the simple universality of the eigenstates does not follow immediately from the structure of the Schrödinger equation (2) with arbitrary $V(y)$.

So far we have merely noted that there exists numbers q which parametrize the various universality classes and scaling regimes. In fact the fluctuation designator is directly related to the values of the critical exponents describing the critical wetting transition in the semi-infinite geometry. This can be seen as follows: in the limit of an infinitely long strip ($M=\infty$) but arbitrary $V(y)$ the ground state wavefunction $\psi_0(y)$ determines the one-body probability distribution: $P(y)=\psi_0^2(y)$. In the limit of $L\to\infty$ (with y fixed) we must recover the correct semi-infinite result for the SDE of $P(y)$ (recall $P(y)/P(1)\propto y^{\theta-1}$ at criticality). From the expression for $S_0(y;q)$ it follows that

$$q=\frac{\theta-1}{2}\qquad\text{all regimes}\tag{13}$$

in agreement with the values quoted above. Recall [8, 9] that θ is itself related to other critical exponents. For instance in the SFL and SFL/WFL regimes (14) further reduces to $q=(\beta_s-1)/2\beta_s$. Substituting in the appropriate critical wetting results for β_s recovers the correct value of the fluctuation designator in these regimes.

Having identified q there still remains the issue of why a simple universal parametrization of the eigenstates should exist. We speculate that this may be a consequence of local scale invariance at fluctuation-dominated wetting transitions in two dimensions. At a wetting transition two dilation factors (see for example [5]) $b_\perp$ and $b_\parallel$ ($=b_\perp^2$ in $d=2$) are needed to specify the global scale invariance of the interfacial fluctuations. Since the renormalization group transformation is a local one, one may conjecture, by analogy with the bulk case, that this scale invariance is obeyed for local perpendicular and parallel dilation factors. Since the dilation factors are necessarily anisotropic the local scale invariance is not conformal. Nevertheless we have argued elsewhere [6] that the local scale invariance of some one-point functions may be described under certain conditions [13] by a conformal mapping. There is a simple reason for this possibility. If a one-point function $f_i(y)$ exhibits a SDE at a wetting transition then this function satisfies the homogeneity condition $f_i(y/b_\perp) = b_\perp^{-(\theta_i - 1)} f_i(y)$. Here θ_i is the generalization of the order parameter SDE critical exponent. In contrast to higher point functions $b_\parallel$ is not required explicitly to specify the global scale invariance of $f_i(y)$ provided the system is translationally invariant in the x direction. The conformal mapping $w = (L/\pi) \ln z$ can be used to map the semi-infinite plane into the infinitely long-strip. Since such a transformation preserves the translational invariance and the capillary-wave form of the Lagrange density [13] we have argued [6, 13] that the behaviour of certain one-point functions in the ($M = \infty$) strip may be correctly described by conformally mapping the SDE law for the semi-infinite plane. We have already shown [6] that this is correct for the one-body probability distribution $P(y)$ which explains why $\psi_0(y)$ is the universal function $S_0(y; q)$ specified by the fluctuation designator q in all regimes. Moreover the universality of $\psi_1(y) \equiv S_1(y; q)$ follows from the observation [13] that the operator $\int_0^y \psi_0(y')\psi_1(y')\,\mathrm{d}y'$ (which corresponds to the (position dependent) amplitude of the asymptotic decay of the connected order-parameter pair correlation function) in the strip is correctly given by conformally mapping its SDE in the semi-infinite plane. Similar remarks apply to the higher excited states [13]. This approach naturally explains why the superuniversality of $S_n(y; q)$ and $\varepsilon_n(q)$ only applies for those potentials (up to an arbitrary additive constant) which are obtained via the logarithmic conformal mapping. Further details on the nature of scale invariance at wetting transitions will appear elsewhere [13].

The author has benefited from conversation with M V Berry, R Evans and J Hannay. This research was supported by the SERC, UK.

References

[1] Fisher M E 1989 *Statistical Mechanics of Membranes and Surfaces* ed D R Nelson, T Piran and S Weinberg (Singapore: World Scientific)

[2] Lipowsky R 1988 *Habilitations-Schrift* (Jülich: Institute für Festkorperforschung) unpublished

[3] Dietrich S 1988 *Phase Transitions and Critical Phenomena* vol 12, ed C Domb and J Lebowitz (New York: Academic) p 1

[4] Privman V and Svrakic N M 1988 *Phys. Rev.* B **37** 3713
Parry A O, Evans R and Nicolaides D B 1991 *Phys. Rev. Lett.* **67** 2978
Privman V and Svrakic N M 1989 *Phys. Rev. Lett.* **62** 633

[5] Lipowsky R and Fisher M E 1986 *Phys. Rev. Lett.* **56** 472; 1987 *Phys. Rev.* B **36** 2126

[6] Parry A O 1992 *J. Phys. A: Math. Gen.* **25** 257

[7] Kroll D M and Lipowsky R 1983 *Phys. Rev.* B **28** 5273

[8] Parry A O 1991 *J. Phys. A: Math. Gen.* **24** 1335

[9] Parry A O 1991 *J. Phys. A: Math. Gen.* **24** L699

[10] Burkhardt T W 1989 *Phys. Rev.* B **40** 6987

[11] Lipowsky R and Nieuwenhuizen T M 1988 *J. Phys. A: Math. Gen.* **21** L89

[12] In the finite width strip with $M = \infty$ and for the present boundary conditions a genuine one-dimensional phase transition takes place as $w \to 3/4^-$. The correlation length diverges like $\xi_\parallel \sim (3/4 - w)^{-1}$ (i.e. $\upsilon = 1$) in this limit.

[13] Parry A O 1992 to appear